

# PARTICLE PRODUCTION IN AN EXPANDING UNIVERSE DOMINATED BY DARK ENERGY FLUID

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## Abstract

We investigate the fate of particle production in an expanding universe dominated by a perfect fluid with equation of state  $p = \alpha\rho$ . The rate of particle production, using the Bogoliubov coefficients, are determined exactly for any value of  $\alpha$  in the case of a flat universe. When the strong energy condition is satisfied, the rate of particle production decreases as time goes on, in agreement to the fact that the four-dimensional curvature decreases with the expansion; the opposite occurs when the strong energy condition is violated. In the phantom case, the rate of particle production diverges in a finite time. This may lead to a backreaction effect, leading to the avoidance of the big rip singularity, specially if  $-1 > \alpha > -\frac{5}{3}$ .

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## 1 Introduction

The phenomenon of particle production in an expanding universe is due, essentially, to the fact that in curved space-time the vacuum is not unique [1, 2]. As the universe evolves, and the curvature changes, the vacuum state also changes: the initial vacuum state, representing the state with no particle, becomes later a multiparticle state. The particle production is directly connected with the curvature of the universe. If the universe is spatially flat, the cosmic evolution leads asymptotically to a Minkowski space-time, where the phenomenon of particle production does not occur anymore. However, this is true only if the strong energy condition is verified: the energy density  $\rho$  and the pressure  $p$  must satisfy the relation  $\rho + 3p > 0$ . If the strong energy condition is violated the particle production should be zero initially, increasing as the universe evolves, in connection with the increase of the four-dimensional curvature with time when  $\rho + 3p < 0$ .

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In order to compute the particle production in a given cosmological model, it is necessary to fix an initial vacuum state. If the universe is always decelerating, there is no natural and unique choice for this, since all modes are initially outside the Hubble radius feeling strongly the curvature of the space-time. However, a natural choice for the initial vacuum state can be done for the case of an accelerating universe, since all physical modes are initially well inside the Hubble radius, and they behave as in a Minkowski space-time. This is one of the reasons why the primordial inflationary scenario is so successful theoretically: it encodes naturally a mechanism for quantum fluctuations, which will be later the seeds for the large scale structure of the universe [3].

Observations indicate that we live today in an inflationary phase, since the universe is accelerating [4, 5]. Hence, the mechanism of particle production may play again a very important role today. Suppose we define a quantum state for a given field today. The question we want to address concerns the rate of particle production. In particular, we would like to know if such particle production is so important that it can alter, by back-reaction, the later evolution of the universe. The back-reaction due to quantum effects has already been studied in the case of the cosmological constant [6]. The back-reaction may be particularly relevant for the case the universe is dominated by a phantom fluid since, classically, in such a situation the universe will inevitably evolve towards a new singular state, called the big rip. We remember that the observational data favors somehow a phantom scenario today [7]. A phantom fluid has many special features. Local configurations of a phantom fluid lead to regular black holes [8]. In cosmology, a universe dominated by a phantom fluid implies a future singularity that will occur in a future finite proper time [9].

We will solve the rate of the particle production when the universe is flat and dominated by an equation of state  $p = \alpha\rho$ , for an arbitrary value of  $\alpha$ . A simplified scenario, not considering the different phase transitions that occur in the real universe, will be adopted. The calculations will be performed for a massless scalar field, for which the corresponding Klein-Gordon equation is solved. Through the quantization of the field, the rate of particle may be determined using the technique of Bogolioubov's coefficients. The general solution for this problem is expressed in terms of Bessel functions. The rate of particle production is determined exactly for any value of  $\alpha$ . The connection of this rate with the strong energy condition is established. It is shown that for a phantom fluid the rate of particle production diverges as the big rip is approached. This may lead to a back-reaction effect changing the effective equation of state of the universe, implying perhaps the avoidance of the big rip itself: the particle produced does not necessarily obey the phantom equation of state and may become the dominant component of the universe if  $-1 > \alpha > -\frac{5}{3}$ . Hence, the back-reaction of the particle production may be an ineluctable mechanism to the avoidance of the big rip if the pressure is not excessively negative. The limiting case,  $\alpha = -\frac{5}{3}$ , has already been found before in a complete different context, that is, in the evolution of classical scalar perturbations in phantom cosmological models [10, 11].

This paper is organized as follows. In the next section we review the formalism of particle creation in a curved space-time. In section 3 we apply the formalism to a perfect fluid cosmological model, and we determine the rate of particle creation for any value of  $\alpha$ . In section 4 we present our conclusions. Even if many steps of the formalism used and of the calculation performed are quite well known, we will present them with some details in order to trace the main features of the final results.

## 2 Basic formalism of particle creation

In the Minkowski spacetime, the Lorentz invariance allows to identify a unique vacuum state. However, in a curved space-time generally the Lorentz symmetries are not present. As a consequence, there is no unique vacuum state, and the particle concept (as excitation of a vacuum state) becomes ambiguous. The nonuniqueness of the vacuum state is responsible for the mechanism of "particle production", which is due, in some sense, to the fact that different observers define different states as the "vacuum state". The way this mechanism operates is quite model dependent. A general way to analyse the problem is to consider a quantum field "living" in a given space-time, and to obtain the particle production via the Bogoliubov transformation that connects different vacuum states in a given model. In the present work we will consider a free, massless scalar field "living" in a flat Friedmann-Robertson-Walker space-time whose dynamics is determined by a barotropic fluid such that  $p = \alpha\rho$ . We will consider  $\alpha$  as an arbitrary parameter. In particular,  $\alpha$  can be negative.

The Lagrangian density describing a massive scalar field  $\varphi$  non-minimally coupled to gravity is given by

$$\mathcal{L}(x) = \frac{1}{2}\sqrt{-g} \left\{ g^{\mu\nu} \varphi(x)_{,\mu} \varphi(x)_{,\nu} - [m^2 + \xi R(x)] \varphi^2(x) \right\} \quad , \quad (1)$$

where  $R$  is the Ricci scalar and  $\xi$  is a coupling constant. The so-called minimal-coupling is obtained when  $\xi = 0$ , while the conformal coupling requires  $\xi = \frac{1}{6}$ . We work in natural unities such that  $G = c = \hbar = 1$ . This Lagrangian density implies the following equation of motion:

$$\square\varphi + m^2\varphi + \xi R\varphi = 0 \quad (2)$$

The scalar product of a pair of solutions of the Klein-Gordon equation (2) is defined by:

$$\left( \varphi_1(x), \varphi_2(x) \right) = i \int_{\Sigma} \left( \varphi_2^*(x) \overleftrightarrow{\partial}_{\mu} \varphi_1(x) \right) [-g_{\Sigma}(x)]^{\frac{1}{2}} d\Sigma^{\mu} \quad (3)$$

where  $d\Sigma^{\mu} = d\Sigma n^{\mu}$  with  $d\Sigma$  being the volume element of the spatial-type hypersurface  $\Sigma$ , and where  $n^{\mu}$  is a timelike unitary vector normal to that hypersurface. This scalar product is independent of the choice of the hypersurface.

We proceed by quantizing the scalar field in the canonical way. The conjugate momentum is defined

by

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \quad , \quad (4)$$

where the dot means derivation with respect to time. The field variable and its conjugate momentum satisfy the usual commutation relations:

$$\begin{aligned} [\varphi(t, x), \varphi(t, x')] &= 0 \quad , \\ [\pi(t, x), \pi(t, x')] &= 0 \quad , \\ [\varphi(t, x), \pi(t, x')] &= i\delta^3(x - x') \quad . \end{aligned} \quad (5)$$

Let us consider  $\{\varphi_j\}$  the complete ensemble of positive solutions of equation (2); so,  $\{\varphi_j^*\}$  will be the complete ensemble of negative solutions of the same equation. The field  $\varphi$  may be written under the form,

$$\varphi = \sum_j (c_j \varphi_j + c_j^\dagger \varphi_j^*) \quad (6)$$

where  $c_j$  and  $c_j^\dagger$  are the annihilation and creation operators such that

$$[c_j, c_j^\dagger] = \delta_{jj'} \quad . \quad (7)$$

The vacuum state  $|0\rangle$  is defined through the condition  $c_j |0\rangle = 0$ . In a flat space-time, the positive solutions are given by positive frequencies, and in this way, considering  $t$  as the time coordinate, a unique vacuum state is obtained, the Minkowski vacuum.

In a curved space-time, the situation is different. In general the choice of  $\{\varphi_j\}$  is not unique, and as consequence the notion of a vacuum state is not unique either. This means that it is not possible to have a universal state describing the absence of particle; the notion of particle itself becomes ambiguous.

Let us consider an asymptotically flat space-time in the past infinity and in the future infinity, but not in the intermediate region. The positive frequency solutions in the past infinity are denoted by  $\varphi_j$ , while the positive frequency solution in the future infinity are denoted by  $\phi_j$ . These solutions obey the following conditions:

$$(\varphi_j, \varphi_{j'}) = (\phi_j, \phi_{j'}) = \delta_{jj'} \quad , \quad (8)$$

$$(\varphi_j^*, \varphi_{j'}^*) = (\varphi_j^*, \varphi_{j'}^*) = -\delta_{jj'} \quad , \quad (9)$$

$$(\varphi_j, \varphi_{j'}^*) = (\phi_j, \phi_{j'}^*) = 0 \quad . \quad (10)$$

The field  $\varphi$  may be expressed, at same time, in terms of the functions  $\{\varphi_j\}$  or in terms of the function  $\{\phi_j\}$ , since both basis are complete. Hence,

$$\varphi = \sum_j (c_j \varphi_j + c_j^\dagger \varphi_j^*) = \sum_j (b_j \phi_j + b_j^\dagger \phi_j^*) \quad . \quad (11)$$

The annihilation and creation operators in the past infinity are given by  $c_j$  and  $c_j^\dagger$ , while  $b_j$  and  $b_j^\dagger$  are the corresponding operators in the future infinity. The vacuum state in the past infinity is defined by  $c_j |0\rangle_{in} = 0 \ \forall j$ , describing the situation where no particle is present initially. The vacuum state in the future infinity is defined by  $b_j |0\rangle_{out} = 0 \ \forall j$ , describing a situation where no particle is present in the future infinity.

The two ensemble of annihilation and creation operators are connected by the relations

$$c_j = \sum_k (\alpha_{kj} b_k + \beta_{kj} b_k^\dagger) \quad , \quad (12)$$

$$c_j^\dagger = \sum_k (\beta_{kj}^* b_k + \alpha_{kj}^* b_k^\dagger) \quad , \quad (13)$$

$$b_k = \sum_j (\alpha_{kj}^* c_j - \beta_{kj}^* c_j^\dagger) \quad , \quad (14)$$

$$b_k^\dagger = \sum_j (\alpha_{kj} c_j^\dagger - \beta_{kj} c_j) \quad . \quad (15)$$

The above relations define the Bogoliubov transformation, where  $\alpha_{jk}$  and  $\beta_{jk}$  are the Bogoliubov's coefficients. Since the asymptotic basis are complete, the in-modes can be expressed in terms of the out-mode and vice-versa:

$$\varphi_j = \sum_k (\alpha_{jk} \phi_k + \beta_{jk} \phi_k^*) \quad , \quad (16)$$

$$\phi_k = \sum_j (\alpha_{jk}^* \varphi_j - \beta_{jk}^* \varphi_j^*) \quad . \quad (17)$$

In order the formalism above to be consistent, the Bogoliubov's coefficients must obey the following relations:

$$\sum_j (\alpha_{jk}^* \alpha_{jj'} - \beta_{jk} \beta_{jj'}^*) = \delta_{kj'} \quad , \quad (18)$$

$$\sum_j (\alpha_{jk}^* \beta_{jj'} - \beta_{jk} \alpha_{jj'}^*) = 0 \quad . \quad (19)$$

Now, we can describe the phenomenon of particle creation by a time-dependent gravitational field. Using the Heisenberg representation,  $|0\rangle_{in}$  is the state system at each moment. The number operator counting the number of particles in the future infinity is  $N_k = b_k^\dagger b_k$ . So, the number of particles in the future infinity is given by:

$$\begin{aligned} \langle N_k \rangle &= \langle 0, in | b_k^\dagger b_k | 0, in \rangle \\ &= \sum_j |\beta_{kj}|^2 \end{aligned} \quad (20)$$

A non null  $\beta_{jk}$  implies particle creation.

### 3 Particle creation in perfect fluid cosmological models

The Robertson-Walker metric describing a four dimensional space-time flat, written in terms of the conformal time, is:

$$ds^2 = a^2(\eta) [d\eta^2 - dx^2 - dy^2 - dz^2] \quad . \quad (21)$$

In terms of the conformal time, the Einstein's equations take the following form:

$$\frac{(a')^2}{a^2} = \frac{8\pi G}{3} \rho a^2 \quad , \quad (22)$$

$$2\frac{a''}{a} - \frac{(a')^2}{a^2} = -8\pi G p a^2 \quad (23)$$

where  $\rho$  and  $p$  are the energy density and the pressure, respectively. We suppose that they are related by a barotropic equation of state such that

$$p = \alpha \rho \quad . \quad (24)$$

The fact that the sound velocity in this fluid must be equal or smaller than the velocity of the light implies that  $\alpha \leq 1$ . From the equations of motion (22,23), we obtain

$$a(\eta) = a_0 |\eta|^\beta \quad , \quad \beta = \frac{2}{1+3\alpha} \quad . \quad (25)$$

If  $\alpha > -1/3$ , the times evolves such that  $0 < \eta < \infty$ , while, for  $\alpha < -1/3$ , the interval is  $-\infty < \eta < 0_-$ . This class of cosmological solutions does not interpolate two minkowskian space-time: the four dimensional curvature diverges for  $\eta \rightarrow 0$  and goes to zero for  $\eta \rightarrow \infty$ . In the region the curvature goes to zero the modes are well inside the horizon and behave as in the Minkowski space-time. This vacuum is called Bunch-Davies vacuum and it plays a fundamental role in the origin of quantum fluctuations in the inflationary scenarios for structure formation. Fixed the initial vacuum state, the formalism described in the preceding section applies for the flat FRW cosmological models and the temporal evolution of the occupation number operator can be computed.

Let us write the solutions of the Klein-Gordon equation under the form

$$\varphi(\eta, \vec{x}) = \int dk^3 \left[ c_{\vec{k}} \varphi_k(\eta, \vec{x}) + c_{\vec{k}}^\dagger \varphi_k^*(\eta, \vec{x}) \right] \quad , \quad (26)$$

where the functions  $\varphi_k(\eta, \vec{x})$ , through a separation of variable, have the form

$$\varphi_k(\eta, \vec{x}) = (2\pi)^{-\frac{3}{2}} e^{i\vec{k}\cdot\vec{x}} \frac{\chi_k(\eta)}{a(\eta)} \quad (27)$$

Using the flat Robertson-Walker metric, the massless Klein-Gordon equation takes the form,

$$\varphi_k'' + 2\frac{a'}{a}\varphi_k + k^2\varphi_k = 0 \quad . \quad (28)$$

Inserting (27) into (28), we obtain

$$\chi_k''(\eta) + \left(k^2 - \frac{a''}{a}\right) \chi_k(\eta) = 0 \quad . \quad (29)$$

Using the expression for the scale factor, the above equation reduces to

$$\chi_k''(\eta) + \left(k^2 - \frac{\beta(\beta-1)}{\eta^2}\right) \chi_k(\eta) = 0 \quad . \quad (30)$$

This is essentially the same equation governing the evolution of gravitational waves in an expanding universe [12].

This equation admits solutions under the form of Hankel's functions:

$$\chi_k(\eta) = \eta^{\frac{1}{2}} \left[ A_k H_q^{(1)}(k|\eta|) + B_k H_q^{(2)}(k|\eta|) \right] \quad , \quad q = \left| \frac{1}{2} - \beta \right| \quad , \quad (31)$$

$H_k^{(1,2)}$  are the Hankel's functions,  $(A_k, B_k)$  are constants whose values are fixed by normalizing the corresponding modes. To do this, we use the Wronskian relation

$$z H_q^{(2)}(z) \partial_z H_q^{(1)}(z) - z H_q^{(1)}(z) \partial_z H_q^{(2)}(z) = \frac{4i}{\pi} \quad . \quad (32)$$

Imposing the orthonormalisation of the modes one obtains:

$$|A_k|^2 = |B_k|^2 = \frac{\pi}{4} \quad . \quad (33)$$

The function  $\chi_k(\eta)$  becomes (up to an arbitrary phase)

$$\chi_k(\eta) = \frac{\sqrt{\pi\eta}}{2} \left( \epsilon_1 H^{(1)}(k|\eta|) + \epsilon_2 H_q^{(2)}(k|\eta|) \right) \quad , \quad \epsilon_{1,2} = 0, 1 \quad . \quad (34)$$

This solution has been already studied previously (see [13]), but in view of a primordial cosmological models, in special in de Sitter or quasi-de Sitter phase.

With the aim of determining the particle production for the cosmological model, we will rewrite the Bogoliubov's transformations in a convenient way. For a minimal coupling and a massless field, the Lagrangian density takes the form:

$$\mathcal{L}(x) = \frac{1}{2} [-g(x)]^{\frac{1}{2}} \left\{ g^{\mu\nu} \varphi(x)_{,\mu} \varphi(x)_{,\nu} \right\} \quad . \quad (35)$$

With the background described above, the Lagrangian can be rewritten as

$$\mathcal{L} = \frac{1}{2} \left\{ \chi'^2 - k^2 \chi^2 - 2 \frac{a'}{a} \chi \chi' + \left( \frac{a'}{a} \right)^2 \chi^2 \right\} \quad . \quad (36)$$

The conjugate momentum is

$$p = \frac{\partial \mathcal{L}}{\partial \chi'} \quad . \quad (37)$$

Hence, the Hamiltonian is

$$\mathcal{H} = \frac{1}{2} \left\{ p^2 + k^2 \chi^2 + 2 \frac{a'}{a} \chi p \right\} . \quad (38)$$

After quantization, the operators  $\mu$  and  $p$  satisfy the commutation relation

$$[\chi, p] = i . \quad (39)$$

We can now determine the expressions for the creation and annihilation operators,  $b^\dagger$  and  $b$  respectively, with respect to the Hamiltonian (38). To do this, we fix:

$$\chi = \frac{1}{\sqrt{2k}}(b + b^\dagger) \quad , \quad p = -i\sqrt{\frac{k}{2}}(b - b^\dagger) . \quad (40)$$

These operators satisfy the commutation relation

$$[b, b^\dagger] = 1 . \quad (41)$$

The Hamiltonian takes the form

$$\mathcal{H} = kb^\dagger b + \sigma(\eta)b^{\dagger 2} + \sigma^*(\eta)b^2 + \frac{i}{2}\left(k + \frac{a'}{a}\right) , \quad (42)$$

where the coupling function is given by  $\sigma(\eta) = ia'/2a$ . Using the normal ordering

$$:\mathcal{H} := \mathcal{H} - \langle 0 | \mathcal{H} | 0 \rangle \quad , \quad (43)$$

one obtains

$$\mathcal{H} = kb^\dagger b + \sigma(\eta)b^{\dagger 2} + \sigma^*(\eta)b^2 . \quad (44)$$

We will use the Heisenberg representation for which the operators evolve with time and the quantum states remain fix. Since there are two independent modes, the Hamiltonian (44) can be written as a sum of two Hamiltonians  $\mathcal{H}_1$  and  $\mathcal{H}_2$  with the same frequency:

$$\mathcal{H}_1 = kb_1^\dagger b_1 + \sigma(\eta)b_1^{\dagger 2} + \sigma^*(\eta)b_1^2 , \quad (45)$$

$$\mathcal{H}_2 = kb_2^\dagger b_2 + \sigma(\eta)b_2^{\dagger 2} + \sigma^*(\eta)b_2^2 . \quad (46)$$

Thus, the Hamiltonian becomes

$$\mathcal{H} = k \left[ b_1^\dagger b_1 + b_2^\dagger b_2 \right] + \sigma(\eta) \left[ b_1^{\dagger 2} + b_2^{\dagger 2} \right] + \sigma^*(\eta) \left[ b_1^2 + b_2^2 \right] . \quad (47)$$

We now define two annihilation operators with momenta  $\vec{k}$  and  $-\vec{k}$ :

$$c_{\vec{k}} = \frac{b_1 - ib_2}{\sqrt{2}} \quad , \quad c_{-\vec{k}} = \frac{b_1 + ib_2}{\sqrt{2}} . \quad (48)$$

The Hamiltonian may be now rewritten in terms of these new operators:

$$\mathcal{H} = kc_{\vec{k}}^\dagger c_{\vec{k}} + kc_{-\vec{k}}^\dagger c_{-\vec{k}} + 2\sigma(\eta)c_{\vec{k}}^\dagger c_{-\vec{k}}^\dagger + 2\sigma^*(\eta)c_{\vec{k}} c_{-\vec{k}} . \quad (49)$$



The new operators obey the following equations:

$$\frac{dc_{\vec{k}}}{d\eta} = -i [c_{\vec{k}}, H] \quad , \quad (50)$$

$$\frac{dc_{\vec{k}}^\dagger}{d\eta} = -i [c_{\vec{k}}^\dagger, H] \quad . \quad (51)$$

Using (49), we find:

$$\frac{dc_{\vec{k}}}{d\eta} = -ikc_{\vec{k}} + \frac{a'}{a} c_{-\vec{k}}^\dagger \quad , \quad \frac{dc_{\vec{k}}^\dagger}{d\eta} = ikc_{\vec{k}}^\dagger + \frac{a'}{a} c_{-\vec{k}} \quad , \quad (52)$$

whose solutions can be written as

$$c_{\vec{k}}^-(\eta) = u_k(\eta)c_{\vec{k}}^-(0) + v_k(\eta)c_{-\vec{k}}^{\dagger}(0) \quad , \quad (53)$$

$$c_{\vec{k}}^{\dagger}(\eta) = u_k^*(\eta)c_{\vec{k}}^{\dagger}(0) + v_k^*(\eta)c_{-\vec{k}}(0) \quad , \quad (54)$$

where  $c_{\vec{k}}^-(0)$  and  $c_{\vec{k}}^{\dagger}(0)$  are the initial values of the operators  $c_{\vec{k}}^-(\eta)$  and  $c_{\vec{k}}^{\dagger}(\eta)$ , respectively. The function  $v_k$  is nothing else than the Bogolioubov coefficient  $\beta_{ij}$  defined before.

From (53), evaluate at  $\eta = 0$ , one obtains:

$$c_{\vec{k}}^-(0) = u_k(0)c_{\vec{k}}^-(0) + v_k(0)c_{-\vec{k}}^{\dagger}(0) \quad , \quad (55)$$

resulting that  $u_k(0) = 1$  and  $v_k(0) = 0$ . Due to the fact that  $[c_{\vec{k}}^-(\eta), c_{\vec{k}}^{\dagger}(\eta)] = 1$ , one can write

$$\left[ u_k(\eta)c_{\vec{k}}^-(0) + v_k(\eta)c_{-\vec{k}}^{\dagger}(0) \quad , \quad u_k^*(\eta)c_{\vec{k}}^{\dagger}(0) + v_k^*(\eta)c_{-\vec{k}}(0) \right] = 1 \quad , \quad (56)$$

resulting that

$$|u_k(\eta)|^2 - |v_k(\eta)|^2 = 1 \quad . \quad (57)$$

That is the normalization condition. We find also the relations

$$\frac{du_k(\eta)}{d\eta} = -ik u_k(\eta) + \frac{a'}{a} v_k^*(\eta) \quad , \quad (58)$$

$$\frac{dv_k(\eta)}{d\eta} = -ik v_k(\eta) + \frac{a'}{a} u_k^*(\eta) \quad (59)$$

The sum  $(u_k + v_k^*)$  satisfies the relation

$$(u_k + v_k^*)'' + \left( k^2 - \frac{a''}{a} \right) (u_k + v_k^*) = 0 \quad . \quad (60)$$

Moreover,

$$\chi_k(\eta) = u_k(\eta) + v_k^*(\eta) \quad , \quad (61)$$

leading to

$$v_k^*(\eta) = \left[ \frac{1}{2} + i \frac{\beta}{2k\eta} \right] \chi_k(\eta) - \frac{i}{2k} \chi_k'(\eta) \quad . \quad (62)$$

Using the solutions (34) for the field  $\chi_k(\eta)$ , with  $\epsilon_2 = 0$ , we find

$$v_k^*(\eta) = \frac{\sqrt{\pi|\eta|}}{4} e^{i\theta_k} \left[ H_q^{(1)}(k|\eta|) - iH_{q-1}^{(1)}(k|\eta|) \right] . \quad (63)$$

Hence,

$$|v_k(\eta)|^2 = \frac{\pi|\eta|}{16} \left[ H_q^{(1)}(k|\eta|)H_q^{(2)}(k|\eta|) + H_{q-1}^{(1)}(k|\eta|)H_{q-1}^{(2)}(k|\eta|) \right. \\ \left. + i \left( H_q^{(1)}(k|\eta|)H_{q-1}^{(2)}(k|\eta|) - H_{q-1}^{(1)}(k|\eta|)H_q^{(2)}(k|\eta|) \right) \right] . \quad (64)$$

This expression gives the rate of particle production. It has the following asymptotic behaviours:

$$k|\eta| \rightarrow 0 \quad , \quad q < \frac{1}{2} \quad (\alpha > \frac{1}{3}) \quad \Rightarrow \quad |v_k(\eta)|^2 \rightarrow \frac{\pi}{8k} \left( \frac{k|\eta|}{2} \right)^{3q-2} ; \quad (65)$$

$$k|\eta| \rightarrow 0 \quad , \quad q = \frac{1}{2} \quad (\alpha = \frac{1}{3}) \quad \Rightarrow \quad |v_k(\eta)|^2 \rightarrow \frac{\pi}{4k} ; \quad (66)$$

$$k|\eta| \rightarrow 0 \quad , \quad q > \frac{1}{2} \quad (\alpha < \frac{1}{3}) \quad \Rightarrow \quad |v_k(\eta)|^2 \rightarrow \frac{\pi}{8k} \left( \frac{k|\eta|}{2} \right)^{1-2q} ; \quad (67)$$

$$k|\eta| \rightarrow \infty \quad , \quad \forall q \quad \Rightarrow \quad |v_k(\eta)|^2 \rightarrow 0 . \quad (68)$$

For  $\alpha > -1/3$ ,  $\eta \rightarrow 0$  implies  $t \rightarrow 0$ , while for  $\alpha < -1/3$ ,  $\eta \rightarrow 0$  implies  $t \rightarrow \infty$ . The asymptotic behavior above just show that, for  $\alpha > -1/3$ , there is no natural initial vacuum state, except for the radiative case  $\alpha = 1/3$ : the initial state is fixed arbitrarily by hand. That is, an decelerating universe requires a previous accelerated phase in order to establishes a natural vacuum initial state. For  $\alpha < -1/3$  there is, in opposition, a natural initial vacuum state, and the particle production is initially zero, becoming infinite asymptotically.

In general, the Hubble radius is given by  $d_H(\eta) \propto \eta^{-\frac{3(1+\alpha)}{1+3\alpha}}$ . The Hubble radius increase with time for  $\alpha > -1$ , remaining constant for  $\alpha = -1$ , and it decreases for  $\alpha < -1$ . Hence, in the phantom case, all modes are well inside the Hubble radius if we go back far enough in the past, and it is quite reasonable to assume that the initial state is the vacuum state <sup>4</sup>. In figure 1 the behavior of the scale factor, of the Hubble radius and of an arbitrary physical model are displayed for a phantom cosmological model, illustrating the situation we have just described.

For the phantom scenario the future asymptotic occurs in a finite proper time. Hence, the particle production may change the final state; in particular, it may lead to the avoidance of the big rip. This may happens since the equation of state of the particles created, for a massless scalar field, is given by  $\rho_s = p_s$ , and we can expect that the phantom fluid will not become the dominant component anymore. In fact, the energy density of perfect fluid scales as  $\rho_{ph} \propto a^{-3(1+\alpha)} \propto \eta^{-\frac{6(1+\alpha)}{1+3\alpha}}$ . If we suppose the energy

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<sup>4</sup>In the real universe we should perhaps prefer to consider an initial non vacuum state, since there was already a particle production in the previous phases of the universe. But, an initial non vacuum state does not change our main conclusions, except for some very special cases.

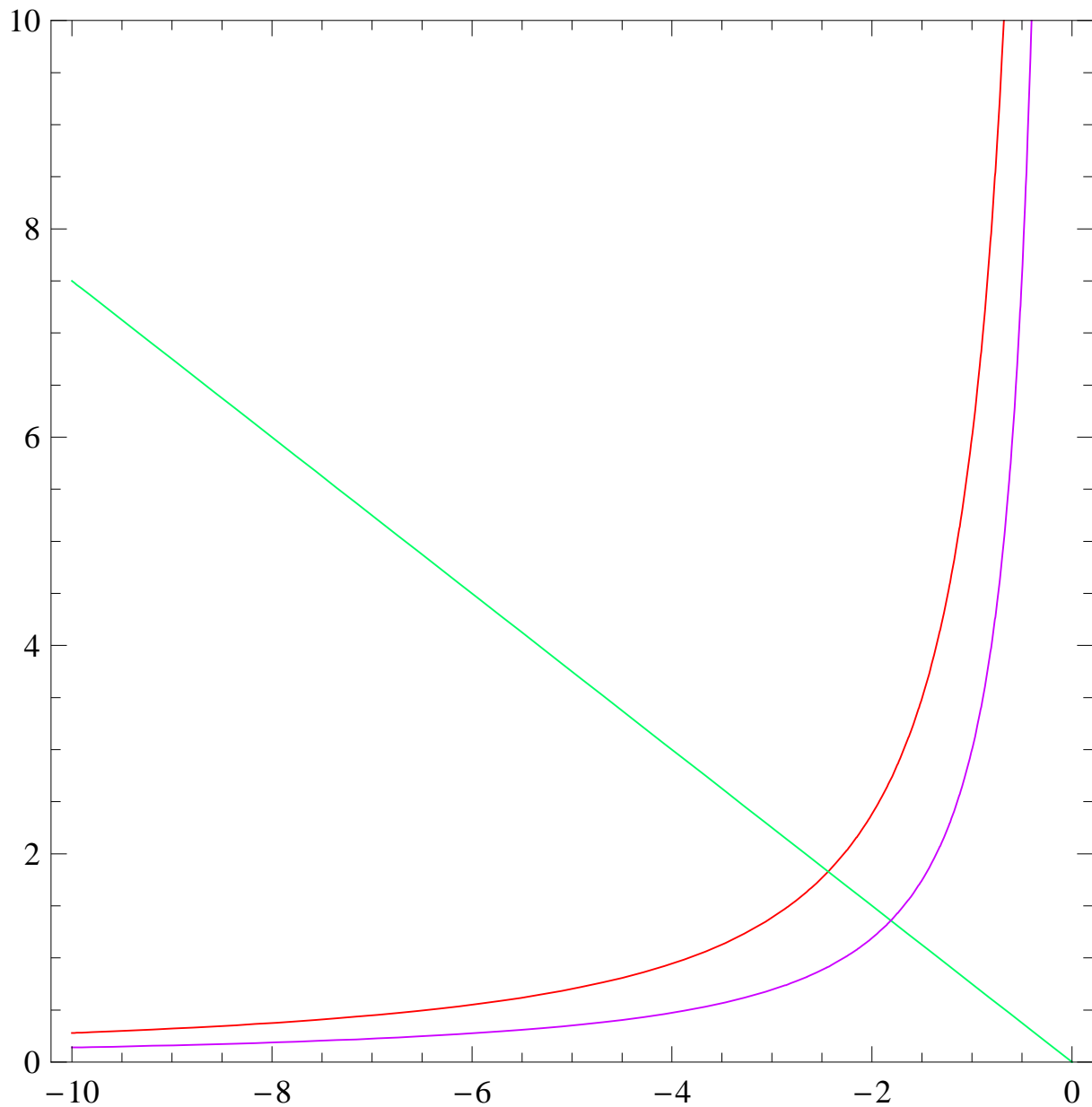


Figure 1: Behavior, as function of time, of the scale factor (red), of the Hubble radius (green) and of a typical physical mode (violet) in a phantom model.

density of the scalar particle created scales according to the rate evaluated above, we have  $\rho_s \propto \eta^{\frac{4}{1+3\alpha}}$ . So the ratio  $\rho_{ph}/\rho_s$  goes to zero if  $-1 > \alpha > -5/3$  and to infinity if  $\alpha < -5/3$ : the back-reaction can be effective if the equation of state of the phantom fluid is not deeply negative.

The particular case  $\alpha = -\frac{1}{3}$  must be studied separately. In terms of the conformal time, the solution

for this case takes the form

$$a(\eta) = a_0 e^{\lambda \eta} \quad . \quad (69)$$

where  $a_0$  and  $\lambda$  are constants. The Klein-Gordon equation now reads,

$$\chi_k''(\eta) + D^2 \chi_k(\eta) = 0 \quad , \quad (70)$$

with

$$D^2 = k^2 - \lambda^2 \quad . \quad (71)$$

There is no propagation of the quantum modes if  $D^2 \leq 0$ . However, for  $D^2 > 0$ , the solution becomes

$$\chi_k(\eta) = \chi_0 e^{-iD\eta} \quad . \quad (72)$$

The normalisation implies that

$$\chi_0 = (2D)^{-\frac{1}{2}} \quad . \quad (73)$$

Using the last expression, we obtain

$$v_k^*(\eta) = \frac{1}{2} \frac{1}{\sqrt{2D}} \left[ 1 - \frac{D}{k} + i \frac{\lambda}{k} \right] \quad . \quad (74)$$

Particles, for this case, are created with a constant rate during all the evolution of the universe for the modes that are inside the Hubble radius, while no particle is created for the modes that are outside the Hubble radius. Note that, in this case, a mode that is initially inside the Hubble radius remains always inside it.

## 4 Conclusions

We have evaluate the rate of creation of massless scalar particle in a universe dominated by a perfect fluid whose equation of state is given by  $p = \alpha \rho$ . An analytic expression has been found in terms of Hankel's functions. Since we have not considered a complete cosmological scenario, with a sequence of different phases, with an initial inflationary phase (as in standard cosmological model), the calculation performed here makes sense, strictly speaking, only when the strong energy condition is violated, that is,  $\alpha < -1/3$ . For such a case, there is a natural initial vacuum state, from which the particle occupation number can be determined. However, we formally extended the calculation for any value of  $\alpha$ . It has been found that for those "dark energy" scenarios the rate of particle creation diverges as  $t \rightarrow \infty$ , that is, in the future infinity.

We have payed special attention to the phantom scenario. The reason is that a universe dominated by a phantom fluid may develop a singularity in a finite future time, the so-called big rip. In this case, we have found that it is possible that the energy density associated to the particles created (which obey

in present case an equation of state of the type  $p_s = \rho_s$ ) becomes dominant over the phantom fluid if  $-1 > \alpha > -5/3$ . Hence, the big rip can be avoided if the pressure is not deeply negative.

It is interesting to remark that a similar critical point,  $\alpha = -5/3$ , has been found in the case of classical scalar perturbation [11]. In that case, however, the evolution of scalar perturbation may destroy the conditions of homogeneity (necessary for the big rip) if  $\alpha < -5/3$ , that is, if the pressure is negative enough. The result found here is exactly the opposite: quantum effects can be operative in the sense of destroying the conditions for the big rip if the pressure is not negative enough. It must be stressed, however, that the evaluation made in the present work must be complemented by a study of more general quantum fields and by a deeper thermodynamical analysis of the energy balance between the phantom fluid and the created particles as the big rip is approached.

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